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# An evaluation of abundance estimates from tagging programs when tag returns are only available from one component of a multi-component fishery: an example based on the 1990's southern bluefin tuna tagging program

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## Abstract

The question of what information can be obtained from tagging experiments when tag return data with reliable reporting rates and catch at age data are available from only one component of a multi-component fishery is examined in the context of SBT fisheries. A Peterson-type mark and recapture estimator of abundance is used as the basis of our examination. The estimator is applied to tag return data from tagging experiments conducted on SBT in the 1990s to examine the type of performance that might be expected from the current CCSBT tagging program, given that reliable reporting rates are unlikely to be available for the longline component of the fishery. The results suggest that using only the SBT tag returns from the surface component of the entire juvenile fishery may still allow for information on juvenile abundances and/or trends, but this requires relatively consistent mixing patterns of tagged fish with the complete population of juvenile fish. It also requires that reliable estimates of reporting rates and of the age distribution of the surface catches are available, which emphasizes the need for developing appropriate statistical estimators for these quantities. The results presented for the 1990s SBT tagging program are reasonably consistent with the assumption of consistent and high levels of mixing. They also indicate no increase (and possibly a decrease) in the strength of cohorts at age 1 during the first half of the 1990s, and suggest a declining trend in abundance by ages 2 and 3 for the surviving members of these cohorts.

## Introduction

Tagging experiments provide a potentially informative approach for reducing uncertainty in stock assessments by providing direct estimates of fishing and natural mortality rates and/or abundance (e.g. Polacheck et al. 1998, 2003; Hoenig et al. 1998; Pollock et al. 2002). This is particularly true for fisheries in which fishery-independent abundance surveys are infeasible and therefore commercial catch rate (CPUE) data must be depended on as the only measure of relative abundance (e.g. in pelagic longline and purse seine fisheries). However, reliable estimation of fishing and natural mortality rates and/or abundance from tagging data requires that estimates of reporting rates be available[1]. Estimation of reporting rates can be problematical for some fishery components in a multi-component fishery. In such cases, can tagging experiments still produce estimates that are useful for stock assessments? The answer to this question is likely to depend upon the nature of the fishery and the other data available from the fishery[2].

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[1] Note if one is willing to assume reporting rates are constant, it is theoretically possible to simultaneously estimate reporting rates as well as fishing and natural mortality rates in a multi-year tagging program of the same cohort. However, the precision of the estimates are generally poor.

[2] For example, if tags are well mixed and there is good information on the catch by age for all fishery components, reporting rates for a missing component can be estimated based on the return rate of tagged fish in that component compared with the return rate of tagged fish in components with reliable reporting rates (e.g. Hearn et al. 2003).

Tagging programs were conducted on juvenile southern bluefin tuna (SBT) in the 1990s in order to provide estimates of fishing mortality and recruitment (e.g. Polacheck et al. 1998). The CCSBT is currently conducting an extensive tagging program with a similar objective (Anon. 2001a), for which tagging commenced in 2001/2002. SBT are harvested by fishing fleets from a number of countries; in particular, juvenile SBT are harvested by Australian purse seiners within the Great Australian Bight (GAB) and by longline fleets from Japan, Korea and Taiwan on the high seas. Observers are currently the only practical and feasible way to obtain estimates of reporting rates from pelagic longline vessels (Polacheck et al. 2004). In the previous 1990s SBT tagging experiments, observers on Japanese longline vessels (principally within Australia's EEZ) were used as a basis for estimating longline reporting rates (Polacheck et al. 1996, 1998). Even so, the estimates of reporting rates had a high degree of uncertainty associated with them (Polacheck et al. 1998). However, the Japanese fleet no longer operates with the Australian EEZ, so this source of observer data no longer exists. In the current situation, obtaining even minimal levels of observer coverage in high-seas longline fisheries has proven to be extremely difficult. Thus, although the CCSBT set a target of 10% observer coverage for all of its major fisheries in 2001 (Anon. 2001b), on the high seas only Japan had placed observers on its vessels in 2002 and the coverage was ~3.5% . No substantive increases were expected in 2003 for any of the fleets (Anon. 2003). The 2003 CCSBT Scientific Committee concluded that the current levels of observer coverage in the Japanese, Korean and Taiwanese longline fleets are not high enough to provide useful estimates of reporting rates from these fleets (Anon. 2003). Thus, reliable estimates of the reporting rates for the main longline fisheries will not be obtainable for at least the first several, if not all, years during which significant tag returns would be expected from the current CCSBT releases.

The current paper examines the question of what information can be obtained from tagging experiments when tag return data with reliable reporting rates and catch at age data are available from only one component of a multi-component fishery. The question is addressed in the context of SBT fisheries, and the approach taken is to consider the tagging and catch at age data in the context of a Peterson-type mark and recapture estimator of abundance (Seber 1973). A Peterson estimator is based on the ratio of the observed number of tags returned within samples taken from the population given the known number of tags released into the population. In a fishery context, the catch at age data constitutes a sample from the population. However, unlike most situations in which a Peterson-type estimator is used, the size of the sample examined for tags is estimated rather than being known exactly. The approach developed is applied to data from tagging experiments conducted on SBT in the 1990s to examine the performance that might be expected from the current CCSBT tagging program, given that the reliable reporting rates are unlikely to be available for the longline component of the fishery.

## Data and Background

Data from the multi-year, multi-age tagging experiments on juvenile SBT conducted in the 1990s (see Polacheck et al. 1998 and references therein) are used here. These experiments tagged fish in Western Australia (WA) and the Great Australian Bight (GAB)[3]. Fish were tagged in WA from 1991 through 1995 and in the GAB from 1991 through 1997. Fish of ages 0 to 2 were tagged in WA, and fish of ages 1 to 5 were tagged in the GAB. Only releases between ages 1 and 2 for WA and ages 1 and 3 for the GAB are considered here since only a small number of releases were outside these age ranges. The age of a fish when tagged was estimated based on its length using cohort slicing and the SBT growth curve currently being used by the CCSBT for its stock assessments. All tagging was done between November and April, so the ages were adjusted in order that fish tagged in November or December from a given year-class/cohort were placed in the same age grouping as those tagged after December. This adjusted age is referred to as a fish's "cohort" age[4], and it is the age used throughout this paper. Table 1 provides the number of tags released by area, cohort and age. The age of the fish tagged in each area reflects the predominant age classes found during tagging in each area.

In this paper, tag returns only from the GAB are considered. Juvenile SBT (ages 1 to 4) tend to spend their summers in coastal waters of Australia, where they are harvested by Australian surface fisheries, and their winters in deeper oceanic waters, where they are harvested by various longline fisheries. Age 0 to 2 fish are found in WA, while age 1 to 4 fish are commonly found in the GAB. The proportion of the global SBT stock for each of these age classes found in WA and the GAB during the summer months is not known; however, it is thought to be a relatively high but diminishing with age. Over the period covered by the

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[3] A relatively small number o fish were tagged in other areas, primarily in waters off eastern Tasmania and from longline vessels. These releases have been excluded from the analyses presented here.

[4] SBT spawn between September and April. For the purpose of aging, all fish are assumed to have a birth date of January 1. Cohort age is defined as its estimated age from cohort slicing (i.e. its calendar age) if a fish was tagged or caught prior to June and as one plus its estimated age from cohort slicing if it was tagged or caught after the end of June.

tagging experiments, the Australian surface fishery shifted from predominately a pole and line fishery targeting fresh SBT for the Japanese sashimi market to a purse seine fishery for tuna farming. Nevertheless, for the fishing seasons of tag recoveries considered here (1991 through 1998) the estimated age composition of the surface catch in terms of cohort ages was relatively consistent, with some shift away from smaller/younger fish (Figure 1).

## Methods

*Basic estimator*

The basic estimator used in this paper is:

$$P^*_{A,a,c,r} = \frac{C_{c,a} N_{A,c,r}}{R_{A,a,c,r}} \tag{1}$$

where:

$A$ =age of tagging;

$a$ = age or ages of recapture (i.e. can be a vector of more than one age);

$c$ = cohort;

$r$ = the region of tagging (WA, GAB, or WA and GAB combined);

$C_{c,a}$ = the catches from cohort $c$ in the GAB at age $a$;

$R_{A,,a,c,r}$ = the number of recaptures in the GAB from cohort $c$ at age $a$ that were released in region $r$ at age $A$;

$N_{A,c,r}$ = the number of releases from cohort $c$ at age $A$ in region $r$;

$P^*_{A,a,c,r}$ = a measure of a cohort $c$'s "strength" based on recaptures at age $a$ from releases at age $A$ in region $r$.

As developed below, alternative interpretations of $P^*_{A,a,c,r}$ are possible depending upon assumptions made about mixing and the proportion of the juvenile stock in the GAB. Note $P^*_{A,a,c,r}$ is only calculated using returns from years after the year of release (i.e. for $a>A$) to allow for heterogeneity in recaptures during the year of tagging (e.g. short-term incomplete mixing during a season within a region; variability in the time of releases relative to the fishery; some releases having occurred in areas near the commercial fishery). In the results presented here tag reporting rates are assumed to be 100%.

*Interpretation of* $P^*_{A,a,c,r}$ *assuming complete mixing*

If there is full mixing of the tagged fish with the untagged fish from a cohort prior to there being any (substantial) differential fishing mortality, then $P^*_{A,a,c,r}$ provides an estimate of the size of a cohort at the age of tagging. This can easily be seen by deriving expressions for the expected catches and number of tag recaptures. Assuming that the fish tagged in a region are a representative sample of the fish in that region and that tagging does not affect their subsequent behaviour or mortality, the expected number of recaptures in the GAB of age *a* fish from cohort *c* that were released at age *A* in region *r* is:

$$R_{A,a,c,r} = \frac{f_{a,c}}{f_{a,c} + m_a} \rho_{A,a,c,r} N_{A,a,c,r} \left(1 - e^{-(f_{a,c}+m_a)}\right) e^{-z^+_{A,a-1,c,r}} \qquad (2)$$

where

$f_{a,c}$ = the fishing mortality rate in the GAB for age *a* fish from cohort *c*;

$m_a$ = the natural mortality rate for age *a* fish (assumed for simplicity to be constant across cohorts);

$\rho_{A,a,c,r}$ = the fraction of fish from cohort *c* that were in region *r* at age *A* that are in the GAB at age *a* (i.e. year *c*+*a*) during the fishing season;

$z^+_{A,a-1,c,r}$ = the cumulative natural and fishing mortality rates between ages *A* and *a*–1 (inclusive) for fish from cohort *c* that were in region *r* at age *A*

Similarly, the number of fish caught in the GAB at age *a* from cohort *c* (i.e. in year *c*+*a*) that were in region *r* at age *A* (i.e. the number of fish caught for which the tagged fish constitute a representative sample) is:

$$C_{A.a,c,r} = \frac{f_{a,c}}{f_{a,c} + m_a} \rho_{A,a,c,r} \varphi_{A,c,r} P_{A,c} \left(1 - e^{-(f_{a,c}+m_a)}\right) e^{-z^+_{A,a-1,c,r}} \qquad (3)$$

where:

$\varphi_{A,c,r}$ = the fraction of cohort *c* that was in region *r* at age *A*;

$P_{A,c}$ = the size of cohort *c* at age *A*;

$C_{A.a,c,r}$ = the catch of age *a* fish in the GAB in year *a*+*c* (from cohort *c*) that were in region *r* at age *A*.

In a parallel manner, the expected catch in the GAB of age *a* fish from cohort *c* that were not in region *r* (referred to as $r^*$) at age *A* is simply:

$$C_{A.a,c,r^*} = \frac{f_{a,c}}{f_{a,c}+m_a} \rho_{A,a,c,r^*} \left(1-\varphi_{A,c,r}\right) P_{A,c} \left(1-e^{-(f_{a,c}+m_a)}\right) e^{-z^+_{A,a-1,c,r^*}} \qquad (4)$$

Adding equations 3 and 4 provides an expression for the total catch in the GAB from a cohort at age *a* (i.e, $C_{c,a}$ from equation 1). Under full mixing and before any differential fishing mortality, the $\rho$ and $z^+$ parameters are equal across regions (i.e. $\rho_{A,a,c,r} = \rho_{A,a,c,r^*}$ and $z^+_{A,a-1,c,r} = z^+_{A,a-1,c,r^*}$), and by dividing the total catch expression (i.e. equation 3 + 4) by equation 2, it is straightforward to show that:

$$\frac{C_{c,a}}{P_{A,c}} = \frac{R_{A,a,c,r}}{N_{A,c,r}} \qquad (5)$$

In other words, $P^*_{A,a,c,r}$ provides an estimate of a cohort's size at the time/age of tagging ($P_{A,c}$). Note that $P_{A,c}$ can be calculated using returns from any age (or set of ages) after the age of tagging, allowing for multiple estimates of a cohort's size at the time of tagging.

If there was a short period of non-mixing (such that natural mortality could be ignored) in which fishing mortality was primarily in the area of releases and the recaptures were known for that period, $P^*_{A,a,c,r}$ could then be used to provide an estimate of a cohort's size at the time after mixing by reducing the number of releases by the number of short term recaptures. For example, in the SBT case, a small number of the GAB releases occurred during the fishing season (rather than at the end of the season) and in areas near the commercial fishery; taking into account the first year's recaptures in the surface fishery could address this problem. Because the numbers of first year recaptures were small, recaptures that occurred during the season of release have not been excluded in the results presented below, but a comparison of the estimates when these recaptures were excluded showed that it had only a minimal effect on the estimates of $P^*_{A,a,c,r}$.

*Interpretation of* $P^*_{A,a,c,r}$ *assuming incomplete mixing*

If there is substantial non-mixing, then the relationship between $P^*_{A,a,c,r}$ and the size of the cohort ($P_{A,c}$) will depend upon: the fraction of the cohort was in the region of tagging at the time of tagging ($\varphi_{A,c,r}$); the relative fraction of the cohort that go into the GAB at age *a* that, at the time of tagging, were in the region of tagging compared to those that were not in the region of tagging (i.e. $\rho_{A,a,c,r}$ compared to $\rho_{A,a,c,r^*}$); and the differential in the fishing mortality rates that the two different groups of fish experience (i.e. $z^+_{A,a-1,c,r}$ compared to $z^+_{A,a-1,c,r^*}$). If all of these are highly variable over time, then estimates of $P^*_{A,a,c,r}$ relative to $P_{A,c}$ will be highly variable and will not provide any useful information about either absolute abundance or trends in abundance. However, there may be some situations in which the ratio $P^*_{A,a,c,r} / P_{A,c}$ will be relatively constant (i.e. a constant relative bias) and thus a time series of $P^*_{A,a,c,r}$ will provide a relative index of $P_{A,c}$.

Appendix 1 provides tables of $P^*_{A,a,c,r} / P_{A,c}$ for a range of values for the parameters $\varphi_{A,c,r}$, $\rho_{A,a,c,r}$, $\rho_{A,a,c,r^*}$, $z^+_{A,a-1,c,r}$ and $z^+_{A,a-1,c,r^*}$. These tables provide an indication of the degree of bias in $P^*_{A,a,c,r}$ that can occur. The bias in $P^*_{A,a,c,r}$ can be either positive or negative and its potential range is quite large (i.e. $P^*_{A,a,c,r} / P_{A,c}$ ranges from 0.31 to 7.76 for the range of the parameter values examined in Appendix 1). As such, without any additional information on the values for the parameters $\varphi_{A,c,r}$, $\rho_{A,a,c,r}$, $\rho_{A,a,c,r^*}$, $z^+_{A,a-1,c,r}$ and $z^+_{A,a-1,c,r^*}$, the absolute values of the estimates for $P^*_{A,a,c,r}$ would be of little value (even as possible bounds for $P_{A,c}$). Examination of the tables in Appendix 1 do suggest that there are some circumstances under which a time series of $P^*_{A,a,c,r}$ could provide a useful relative index of $P_{A,c}$. This would clearly apply if $\varphi_{A,c,r}$, $\rho_{A,a,c,r}$, $\rho_{A,a,c,r^*}$, $z^+_{A,a-1,c,r}$ and $z^+_{A,a-1,c,r^*}$ were constant over time. However, there appear to be some other situations in which the variability in the relative bias would be expected to remain relatively small. For example, if $\varphi_{A,c,r}$ and $\rho_{A,a,c,r}$ are reasonably high and consistent (i.e. a large fraction of a cohort at the age of tagging is available for tagging and the proportion of these fish that go into to the area of recapture is high) combined with low rates of exploitation prior to the recovery for the tagged component of the population (conditions which are thought to apply to 1-year-old SBT), then $P^*_{A,a,c,r}/P_{A,c}$ would appear relatively insensitive to reasonable variation in $\rho_{A,a,c,r^*}$ and $z^+_{A,a-1,c,r^*}$.

It is also worth noting that if $\rho_{A,a,c,r}$ and $z^+_{A,a-1,c,r}$ are relatively constant, then $P^*_{A,a,c,r}$ would provide a relative abundance index of the number of age *a* fish from a cohort in the area of recapture. In the case of SBT, given that fishing mortality rates on age 1 are near zero, a time series of estimates of $P^*_{A,a,c,r}$ from returns at age 2 would provide a good measure of relative abundance of age 2 fish in the GAB as long as $\rho_{A,a,c,r}$ was relatively constant (i.e. as long as in each year the same proportion of age 1 fish that were in the region of tagging go into the GAB at age 2). In this situation, the data provide a straightforward Peterson estimate except that the number of marked fish available for recapture is unknown but has been proportionally reduced by the same fraction in each year.

### *Variance and confidence intervals for* $P^*_{A,a,c,r}$

There are two principle sources of errors in the estimates of $P^*_{A,a,c,r}$ (assuming 100% reporting rates). The first is the sampling error associated with the number of tags recaptured and the second is estimation error associated with the number of fish caught during a year. There is also potential error in the actual number of tags released by age from a cohort due to aging errors in the cohort sliced age estimates. This latter source of error is not considered here.

Tag return data are commonly modelled as multinomial. However, there is a number of factors that lead to recaptures being over-dispersed relative to a multinomial (e.g. heterogeneity in recapture probabilities as a result of schooling behaviour, or variability in selectivity among vessels). Previous analyses of these tagging data indicated over-dispersion in the data exists and a bootstrap approach was suggested as a mechanism to estimate the variance associated with sampling error in the number of recovered tags (Polacheck et al. 1998). Thus, a bootstrap approach was used here. Bootstrap samples were constructed by randomly selecting, with replacement, days from which tag releases occurred during a release season and area. Days were selected until the bootstrap sample comprised the same number of days for which tagging actually occurred. All tags released on the selected days and their associated recoveries were included in the bootstrap sample. The bootstrap samples were thus conditioned on the number of days for which tagging occurred; the actual number of tags included in different bootstrap samples varied.

The appropriate approach to estimate the error associated with the catch at age estimates is less clear. The estimates of the catch at age are derived from a complex system of sampling the catch for length, converting the sampled length measurements to age estimates using cohort slicing, then scaling up the estimated sample age distribution using estimates of the total catch. Total catch in numbers is estimated from the landed weight of the catch divided by the estimated mean weight in the case of pole and line caught fish, and from video counts of fish during transfers from towing to farm cages in the case of the purse seine caught fish. For the purpose of getting an indication of how errors in the estimated catch at age data contribute to the error in the estimates of $P^*_{A,a,c,r}$, a Monte-Carlo approach was taken. In this approach, it was assumed that the major source of error in the catch at age estimates comes from estimating the age distribution; the total catch within a fishing season was assumed to be known exactly. Monte-Carlo re-sampling of the catch at age estimates was then performed assuming multinomial sampling with a pre-specified effective sample size. A sample size of 50 was used in the results presented here.

A single bootstrap sample was combined with one realization from the Monte-Carlo re-sampling of the catch at age data to produce a single “bootstrap/Monte-Carlo” estimate of $P^*_{A,a,c,r}$. This process was repeated 1000 times to derive an estimate of the coefficient of variation (CV) and confidence interval for $P^*_{A,a,c,r}$. Note that in some cases a bootstrap sample could contain a set of releases for which there were zero recaptures for the age range of recaptures being considered. In such cases, it was not possible to estimate $P^*_{A,a,c,r}$, and such a bootstrap sample was excluded in the calculation of the bootstrap mean, variance and confidence interval for that age range. This will underestimate the overall uncertainty. The bootstrap/Monte-Carlo estimates of the CVs and confidence intervals presented here are clearly conditional on the assumed effective sample size for the catch at age data, but they provide both an indication of what these values might be and the relative contribution of the error associated with the tagging and catch at age data in the overall estimates of $P^{*}_{A,a,c,r}$.

## Results

Figure 2 compares estimates of $P^{*}_{A,a,c,r}$ based on returns at age 2, 3, 4 and 2-4 pooled for fish released at age 1 in WA. With the exception of the estimate based on returns of age 2 fish for the 1994 cohort, the estimates based on the returns for different ages exhibit a large degree of consistency within a cohort (e.g. the 90% confidence error bars for the age-specific return estimates overlap with the pooled age estimates). Such agreement is consistent with full and complete mixing of the tagged fish with all the fish from a cohort. In the case of the 1994 cohort, the large difference stems from very few tagged fish having been returned at age 2 relative to the estimated number of age 2 fish caught, although the very wide error bars for this estimate indicate that there is large uncertainty associated with it. Figure 3 presents similar estimates as those in Figure 2 except the estimates are based on age 2 releases in WA. Figures 4 to 6 present similar results for releases in the GAB at age 1, 2 and 3 respectively. In general, the estimates based on returns at different ages tended to be relatively consistent within a cohort, although fewer actual comparisons could be made in some of these figures (e.g. Figure 3 is sparse because relatively few age 2 fish are tagged in WA; in Figure 6, which shows results for age 3 releases in the GAB, estimates are only possible using age 4 returns). The largest discrepancy in the age-specific estimates in these later figures is between the 1991 cohort estimates using age 3 versus age 4 returns from releases at age 2 in the GAB (Figure 5). In this case, it is not possible to distinguish which is the more likely "outlier". In this context, the estimate for the 1991 cohort using age 3 releases (based on age 4 returns) in Figure 6 also appears to be low based on the time trend in the estimates. It is perhaps worth considering that the 1991 cohort estimates based on age 4 returns (from age 2 and 3 GAB releases) and the 1994 cohort estimates based on age 2 returns (from age 1 WA releases) are dependent upon the estimates of the catch at age for the 1994/1995 and 1995/1996 fishing seasons. Sampling protocols to deal with the increasing farm catches were developing during this period. As such, the age compositional data from this period may be less reliable and warrant further examination.

Figure 7 compares the estimates of $P^{*}_{A,a,c,r}$ using age 1 releases from WA, the GAB, and both areas combined. Figure 8 shows similar results but for age 2 releases. In both cases, only the estimates based on the pooled tag return data up through age 4 are shown. For both age 1 and 2 releases, the estimates of $P^*_{A,a,c,r}$ using WA and GAB releases appear reasonably consistent. This could be considered further indication of a high degree of mixing of tagged fish with all the fish from a cohort. The estimates of $P^*_{A,a,c,r}$ for the combined WA and GAB release data tend to follow closely the estimates for either the WA estimates (in the case of age 1 releases) or the GAB estimates (in the case of age 2 releases). This reflects the fact that releases from WA tend to dominate the overall age 1 releases, while GAB releases tend to dominate the age 2 releases for each cohort (Table 1). This simply reflects the areas where these age-classes of fish are generally found.

A further check of the consistency of the $P^*_{A,a,c,r}$ estimates in terms of mixing and as a possible measure of a cohort’s absolute abundance at the age of tagging is to compare the estimates for each cohort across release ages. If the estimates represent estimates of absolute abundance, then the estimates at each successive release age should decrease, reflecting the natural and fishing mortality that occurred on each cohort. Figure 9 compares the estimates of abundance for ages 1 and 2 based on the combined releases from WA and the GAB and returns pooled across all recapture ages through age 4. Figure 10 provides a similar comparison but for age 1, 2 and 3 estimates based on releases from the GAB only. In the case of the releases from the GAB (Figure 10), the estimates for a cohort always decrease with age and the magnitude would appear reasonable. However, this would best be evaluated within an overall stock assessment that considered all catch from a cohort. In the case of the estimates from the combined releases (Figure 9), the estimates of age 1 abundance are generally greater than the age 2 estimates. However, in three of the six comparisons[5], the differences appear to be relatively small if natural mortality rates are considered. For age 1 SBT, natural mortality rates have been estimated to be relatively high (~0.3-0.5) based on alternate analyses of these same tagging data (Polacheck et al. 1998). As noted above, the estimates for the combined age 1 releases are dominated by releases from WA (with the exception of the 1995 and 1996 cohorts), while the age 2 releases are dominated by releases from the GAB. This would suggest that, to the extent that these combined estimates for ages 1 and 2 are considered inconsistent with an assumption of complete mixing, a greater fraction of the stock of age 1 fish represented by the tagged fish in WA ends up in the GAB than the fraction of the stock

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[5] Note that the 1995 age 1 estimate is based on only 82 releases from the GAB (as there were no WA releases in this year) and 15 returned tags (see Tables 1 and 2), and perhaps should be excluded in this comparison.

of age 2 fish represented by the tagged fish in the GAB (i.e. too many age 1 WA releases were recaptured relative to age 2 GAB releases).

## Discussion

The estimates of $P^*_{A,a,c,r}$ presented here suggest that using only the SBT tag returns from the surface component of the entire juvenile fishery may still allow for information on juvenile abundances and/or trends. This requires relatively consistent mixing patterns of tag fished with the complete population of juvenile fish. From the results presented for the 1990's SBT tagging program, the estimates of $P^*_{A,a,c,r}$ for a cohort at a particular age and based on a particular area of release (e.g. WA or GAB) are relatively consistent using returns at different ages, suggesting relatively consistent, if not complete mixing. Comparisons of the estimates for different ages of release also suggest a reasonable degree of consistency in mixing for releases at age 1, 2 and 3 from the GAB, but some concerns about lack of complete mixing within the juvenile population when estimates based on combined releases from WA and GAB are considered. The results suggest that possibly too high of a proportion of age 1 releases (particularly from WA) are subsequently recaptured in the GAB relative to age 2 releases. To the extent that the estimates presented here from the 1990s tagging experiments are considered to provide information on juvenile abundances, the results provide no indication of an increase (and possibly a decrease) in the strength for cohorts at age 1 from the first half of the 1990s, and suggest a declining trend by age 2 and 3 in abundance for the surviving members from these cohorts.

It should be emphasised that the estimates using only tag return data from the Australian fishery as presented here require that reliable estimates of reporting rate and of the age distribution of the surface catches are available. As such, the level of precision and potential biases in the resulting estimates of $P^*_{A,a,c,r}$ will be critically dependent upon the level of sampling for lengths in the surface fishery and the accuracy with which these are converted to age frequency estimates for the entire catch. The estimates of the CVs presented here for $P^*_{A,a,c,r}$ are only indicative as they are based on a rather arbitrary assumption of multinomial sampling error for the catch at age data with an effective sample size of 50. In most cases, the error in the catch at age tends to be an equal or dominant contributor (compared to the error in the tag returns) to the estimates of the CVs (Table 2). Thus, substantial reduction in the effective sample sizes in these calculations would, in most cases, result in substantial increases in the CVs of the estimates. If meaningful estimates of precision are to be derived from these tagging experiments, it is critical that appropriate statistical models be developed for the actual estimation procedure for the catch at age data. Similarly, the estimates of the tag reporting rates in the surface fishery are critical. The CV associated with these could be a substantial contributor to the overall precision of the estimates if not estimated with reasonable precision. Moreover, any bias in the reporting rate would act as a multiplicative bias on $P^*_{A,a,c,r}$ .

It should be noted that with the assumption of complete mixing and the existence of reporting rates estimates for the surface fishery, the tag returns from the surface fishery could be used along with estimates of the catch at age in both the surface and longline fisheries to estimate the reporting rates in the longline fisheries (see Hearn et al. 2003). The tag return data could then be used in a Brownie model estimation framework as envisioned in the original design of the SRP tagging program. Incorporation of any reported tags from the longline fishery would potentially add some, but little, information to the overall estimates (e.g. in a Brownie estimation framework, estimates of total mortality do not require estimates of reporting rates). However, the precision and accuracy of the mortality estimates would be highly dependent upon the variance and potential biases in the estimated catch at age data from the longline fisheries, since the estimates of the reporting rates are directly dependent upon these data. In addition to this dependence on the catch at age estimates from the longline fishery, a major disadvantage of using such a Brownie approach as compared to the Peterson-type approach used here is that it would provide little scope for diagnostics and testing assumptions of the underlying model. Moreover, the biases in the mortality rate estimates from any incomplete mixing would tend to be amplified by estimating the longline reporting rates this way[6].

Fully appropriate direct incorporation of the estimates of $P^*_{A,a,c,r}$ into a statistical catch at age stock assessment model similar to those being used for SBT would be problematical because of the need to account for the double use of the catch at age data (i.e. once in the estimates of $P^*_{A,a,c,r}$ and once as a directly fitted component in the assessment). Alternatively, the tag

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[6] For example, in a case with low mixing into the areas of the longline fishery, a low number of tag returns from the longline fishery relative to the number of fish caught would incorrectly be considered to represent low reporting rates. Thus, the estimated number of actual recaptures would be too high, which would further bias upward the estimates of fishing mortality rates.

return data could be incorporated by using the stock assessment model to predict the expected number of returns from the Australian surface fishery conditional on the rest of the model structure and parameters (this would be similar to the approach currently being used in the stock assessment models and, functionally, would be more similar to a Brownie model approach). However, developing an appropriate likelihood for the tag return component (and thus determining the appropriate weight to be given to the tag data in the overall estimation) would be challenging.

Finally, one advantage of using the approach developed here when there is little or no information on the reporting rates from the longline fisheries is that it can provide an indicator of trends in juvenile abundances over the period of the tagging experiments independent of any assumptions about tag returns and catches in the longline fishery. Such indicators can provide a useful independent check on overall complex stock assessment results.

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Table 1: Number of tag releases by cohort, area and cohort age. Note a small number of tags estimated to be age 3 were released in WA (88), and a small number of tags estimated be age 0, 4 or 5 were released in the GAB (291).

|  | *WA* |  | *GAB* |  |  |
|---|---|---|---|---|---|
| *Cohort* | 1 | 2 | 1 | 2 | 3 |
| 1988 | 0 | 0 | 0 | 0 | 810 |
| 1989 | 0 | 354 | 0 | 2773 | 1096 |
| 1990 | 2645 | 891 | 654 | 3755 | 2692 |
| 1991 | 2111 | 289 | 33 | 2648 | 3640 |
| 1992 | 4522 | 49 | 376 | 3109 | 2627 |
| 1993 | 8442 | 1756 | 561 | 4143 | 1511 |
| 1994 | 8170 | 0 | 415 | 2518 | 526 |
| 1995 | 0 | 0 | 82 | 592 | 0 |
| 1996 | 0 | 0 | 884 | 0 | 0 |

Table 2: Comparison of the bootstrap/Monte-Carlo CV estimates for $P^*_{A,a,c,r}$ when the estimates are based on: the bootstrap component for the tagging data only; the Monte-Carlo component for the catch-at-age data only; and both. R is the total number of tags recovered from a cohort for ages 2 to 4 in the case of age 1 releases, and for ages 3 to 4 in the case of age 2 releases.

|  |  |  |  | CV |  |  |
|---|---|---|---|---|---|---|
| Release age | Cohort | $P^*_{A,a,c,r}$ | R | tag & catch | tag only | catch only |
| 1 | 1990 | 5.18 | 57 | 0.27 | 0.24 | 0.11 |
|  | 1991 | 2.89 | 75 | 0.14 | 0.10 | 0.10 |
|  | 1992 | 2.24 | 281 | 0.17 | 0.12 | 0.12 |
|  | 1993 | 2.28 | 782 | 0.12 | 0.06 | 0.11 |
|  | 1994 | 2.19 | 820 | 0.12 | 0.04 | 0.11 |
|  | 1995 | 1.18 | 15 | 0.37 | 0.36 | 0.11 |
|  | 1996 | 3.70 | 80 | 0.29 | 0.28 | 0.08 |
| 2 | 1989 | 2.17 | 104 | 0.17 | 0.13 | 0.11 |
|  | 1990 | 2.53 | 127 | 0.13 | 0.06 | 0.11 |
|  | 1991 | 2.86 | 80 | 0.14 | 0.09 | 0.11 |
|  | 1992 | 1.91 | 203 | 0.14 | 0.07 | 0.12 |
|  | 1993 | 1.49 | 665 | 0.13 | 0.06 | 0.12 |
|  | 1994 | 1.00 | 425 | 0.12 | 0.04 | 0.12 |
|  | 1995 | 1.01 | 93 | 0.12 | 0.04 | 0.11 |

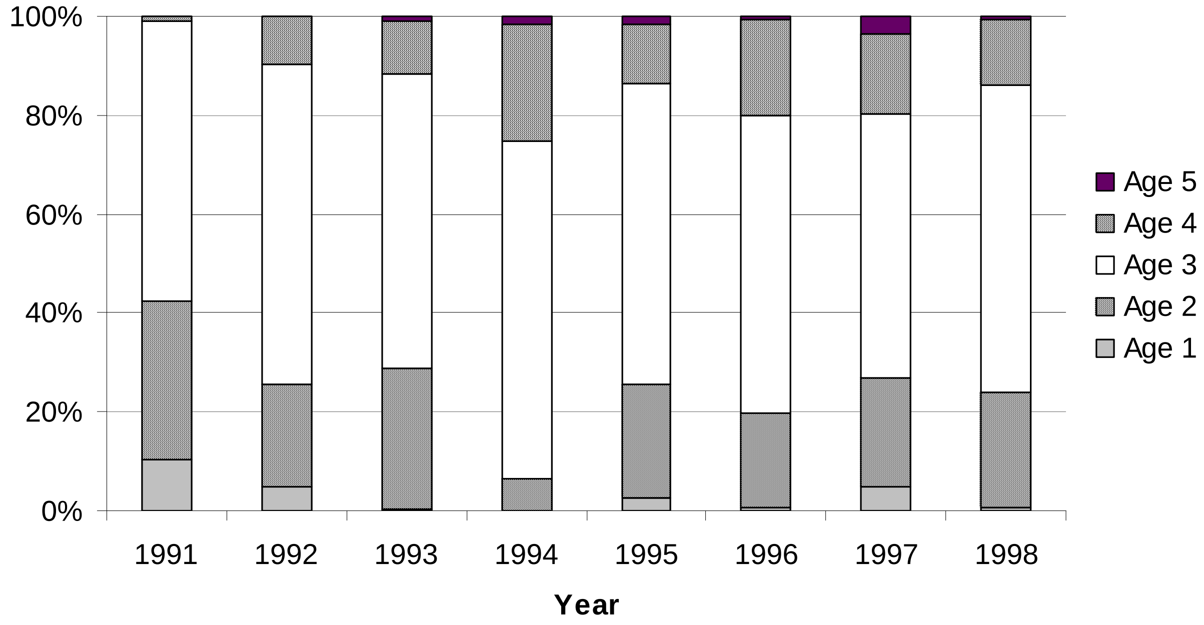

Figure 1: Estimated age composition of Australian surface fishery catches in the Great Australian Bight by fishing season (e.g. 1991 refers to the 1990/1991 fishing season).

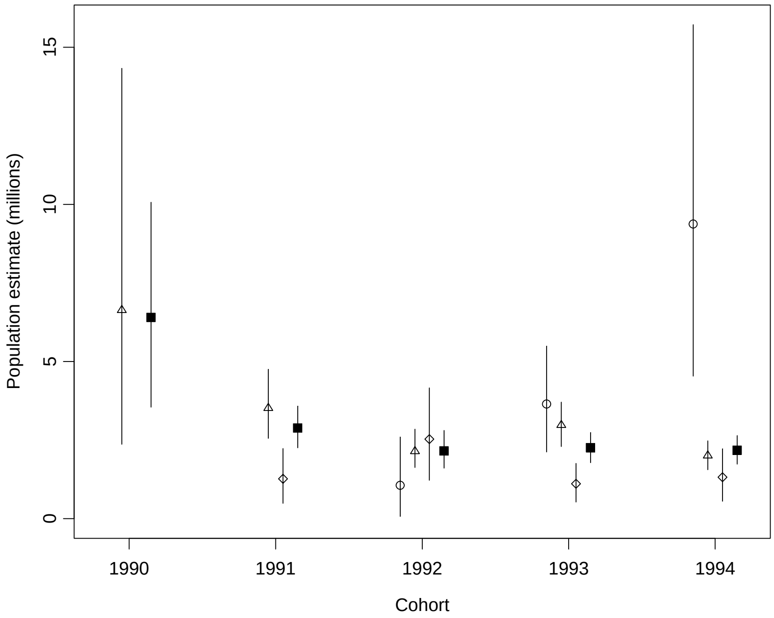

Figure 2: Peterson estimates for the number of 1-year-old SBT ( $P^*_{A,a,c,r}$ ) based on age 1 releases from Western Australia. Circles represent estimates based on age 2 returns, triangles on age 3 returns, diamonds on age 4 returns, and solid squares on returns from ages 2-4 pooled. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown.

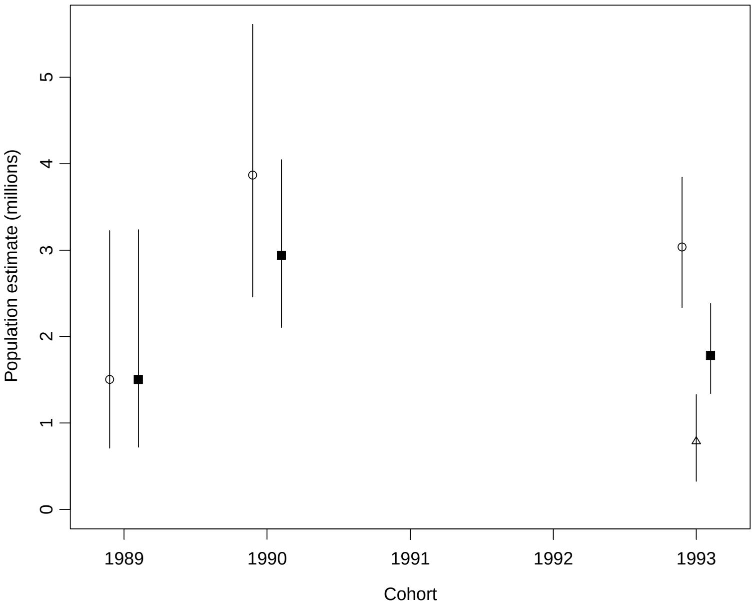

Figure 3: Peterson estimates for the number of 2-year-old SBT ( $P^*_{A,a,c,r}$ ) based on age 2 releases from Western Australia. Circles represent estimates based on age 3 returns, triangles on age 4 returns, and solid squares on returns from ages 3 and 4 pooled. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown.

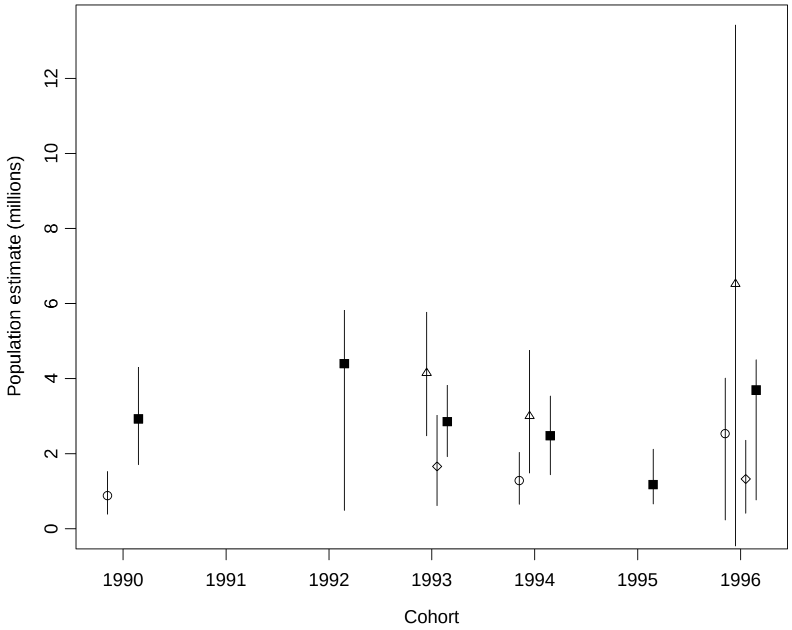

Figure 4: Peterson estimates for the number of 1-year-old SBT ( $P^{*}_{A,a,c,r}$ ) based on age 1 releases from the Great Australian Bight. Circles represent estimates based on age 2 returns, triangles on age 3 returns, diamonds on age 4 returns, and solid squares on returns from ages 2-4 pooled. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown.

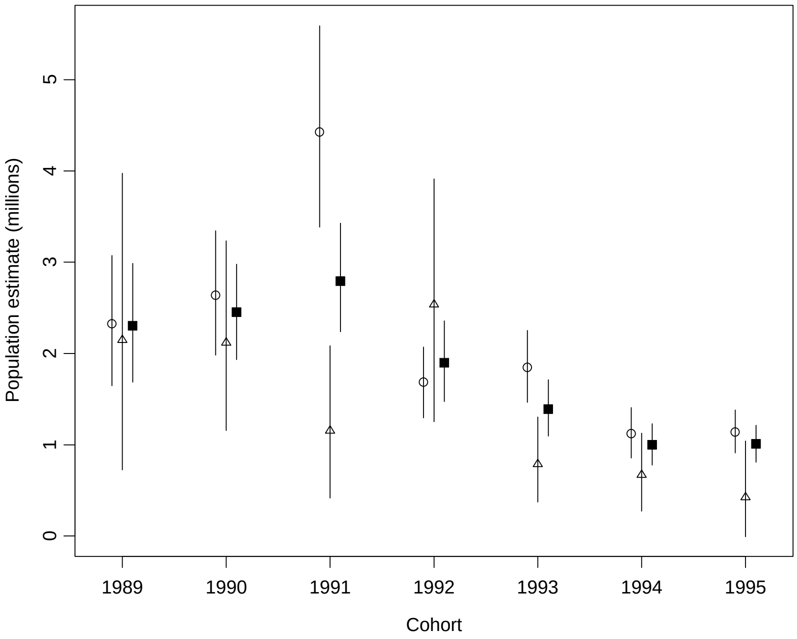

Figure 5: Peterson estimates for the number of 2-year-old SBT ( $P^{*}_{A,a,c,r}$ ) based on age 2 releases from the Great Australian Bight. Circles represent estimates based on age 3 returns, triangles on age 4 returns, and solid squares on returns from ages 3 and 4 pooled. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown.

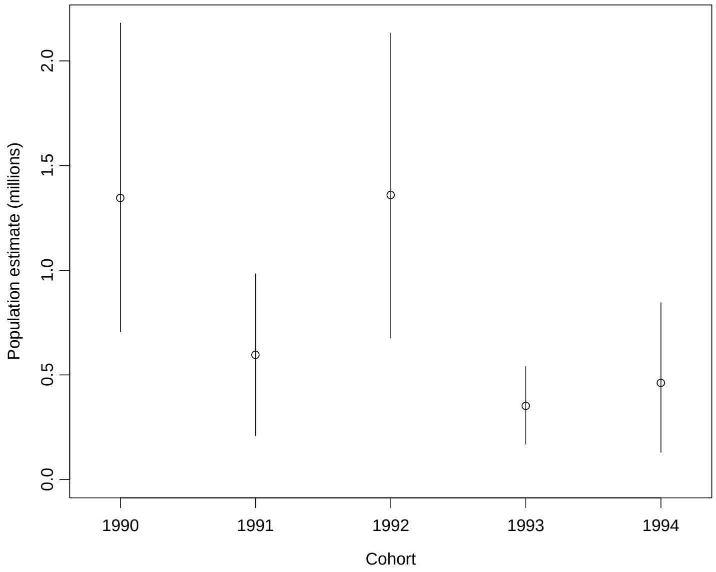

Figure 6: Peterson estimates for the number of 3-year-old SBT ( $P^{*}_{A,a,c,r}$ ) based on age 3 releases from the Great Australian Bight. The estimates are based on age 4 returns. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown.

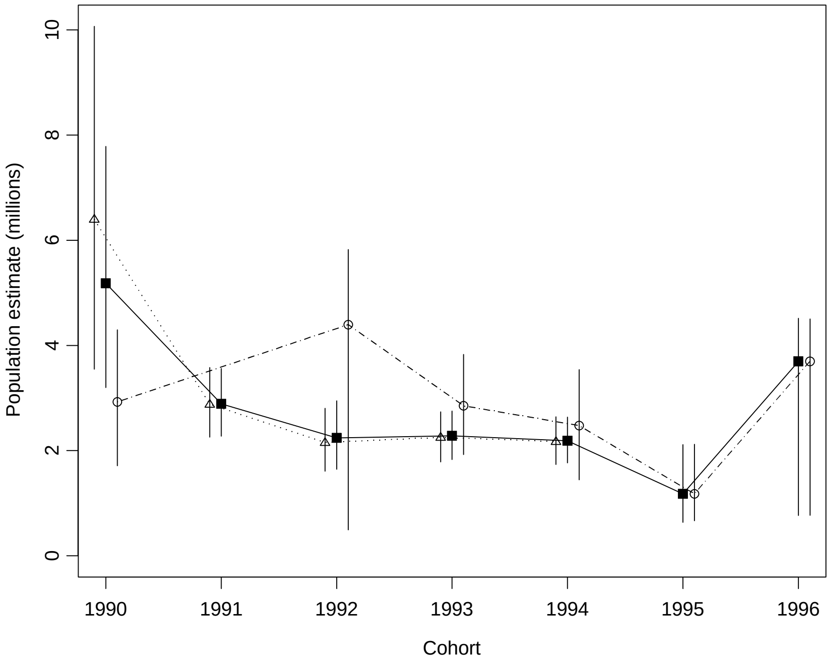

Figure 7: Comparison of Peterson estimates for the number of 1-year-old SBT ( $P^{*}_{A,a,c,r}$ ) based on age 1 releases from Western Australia (triangles), the Great Australian Bight (circles), and both areas combined (solid squares). The estimates shown are based on the pooled returns and catches for ages 2 to 4. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown. Note that slight differences in the confidence intervals when there were only releases in one area (1995 and 1996) represent different realizations of 1000 bootstrap/Monte-Carlo estimates

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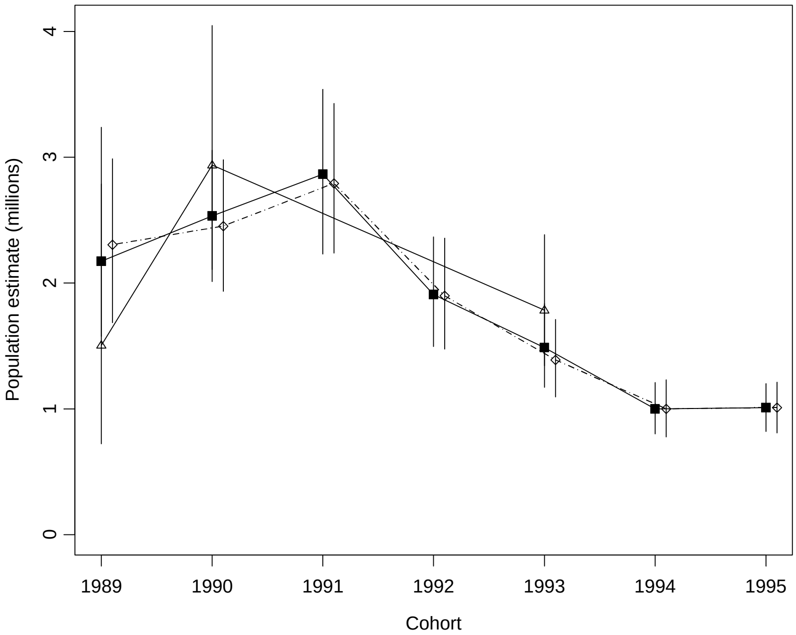

Figure 8: Comparison of Peterson estimates for the number of 2-year-old SBT ( $P^{*}_{A,a,c,r}$ ) based on age 2 releases from Western Australia (triangles), the Great Australian Bight (circles), and both areas combined (solid squares). The estimates shown are based on the pooled returns and catches for ages 3 and 4. Error bars are estimated 90% confidence intervals (see text). Only estimates based on more than 10 returns are shown. Note that slight differences in the confidence intervals when there were only releases in one area (1994 and 1995) represent different realizations of 1000 bootstrap/Monte-Carlo estimates.

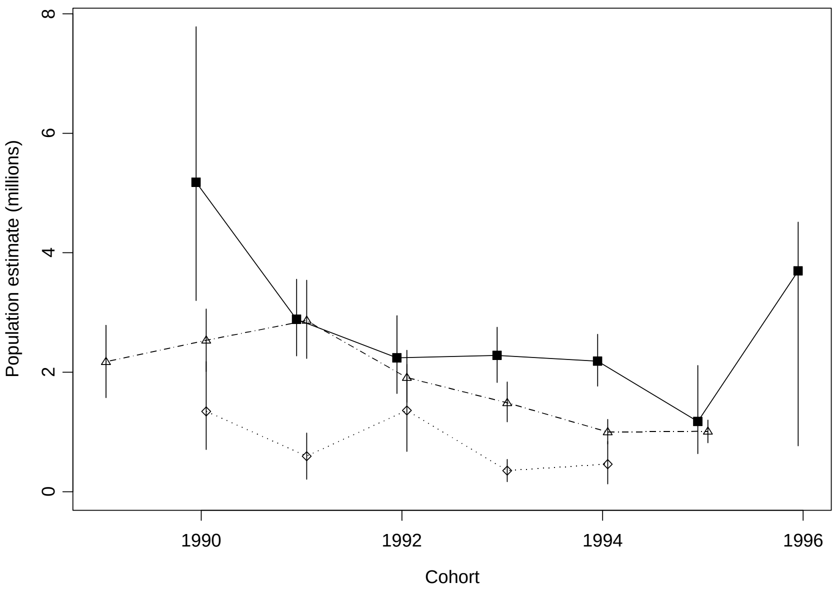

Figure 9: Comparison of Peterson estimates for the number of 1-, 2- and 3-year-old SBT ($P^*_{A,a,c,r}$) based on age 1 releases (solid squares), age 2 releases (triangles) and age 3 releases (diamonds), respectively, from Western Australia and the Great Australian Bight combined. Note there were no age 3 releases in Western Australia. The estimates shown are based on the pooled returns and catches for each age of release. Error bars are estimated 90% confidence intervals (see text).

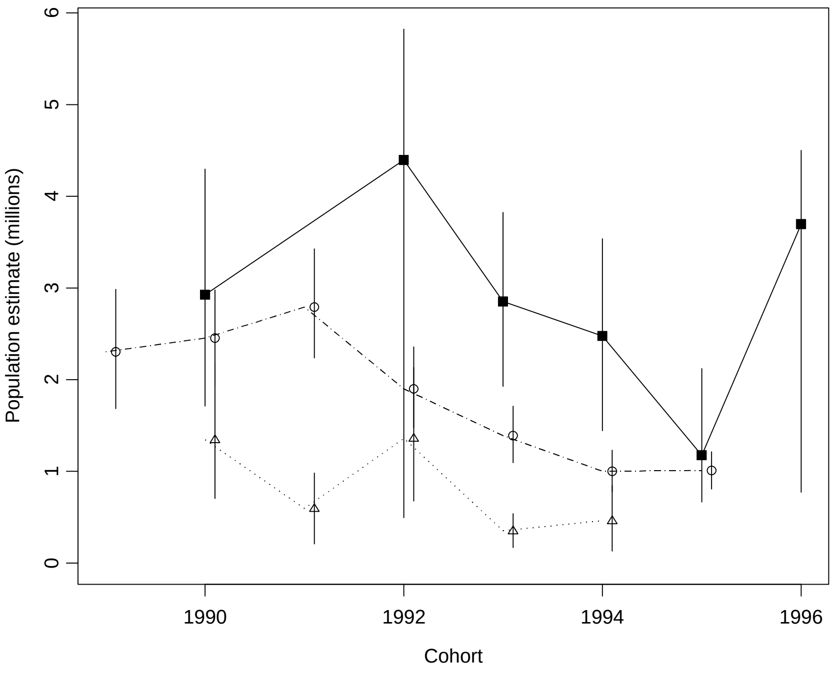

Figure 10: Comparison of Peterson estimates for the number of 1-, 2- and 3-year-old SBT ($P^*_{A,a,c,r}$) based on age 1 releases (solid squares), age 2 releases (circles), and age 3 releases (triangles), respectively, from the Great Australian Bight only. The estimates shown are based on the pooled returns and catches for each age of release. Error bars are estimated 90% confidence intervals (see text).

## Appendix 1

Tables of values for $P^*_{A,a,c,r} / P_{A,c}$ for a range of values for the parameters $\varphi_{A,c,r}$, $\rho_{A,a,c,r}$, $\rho_{A,a,c,r^*}$, $z^+_{A,a-1,c,r}$ and $z^+_{A,a-1,c,r^*}$. Note that in the results presented here $z^+_{A,a-1,c,r}$ and $z^+_{A,a-1,c,r^*}$ have been separated into a fishing and natural mortality component with natural morality assumed to be independent of region *r*. Thus,

$$z^+_{A,a-1,c,r} = f^+_{A,a-1,c,r} + m^+_{A,a-1,c} \quad \text{and}$$

$$z^+_{A,a-1,c,r^*} = f^+_{A,a-1,c,r^*} + m^+_{A,a-1,c}$$

where *f* and *m* refer to fishing and natural mortality rates respectively. In the results presented in this appendix $m^+_{A,a-1,c}$ has been fixed at 0.35.

Table 1: Comparison of the ratio of $P^{*}_{A,a,c,r}/P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $f^{+}_{A,a-1,c,r^*}$ when $\varphi_{A,c,r}$ is fixed at 0.8, $\rho_{A,a,c,r}$ is fixed at 0.8 and $f^{+}_{A,a-1,c,r}$ is fixed at 0.1. Note $m^{+}_{A,a-1,c}$ = 0.35.

| $\rho_{A,a,c,r^*}$ | $f^{+}_{A,a-1,c,r^*}$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 |
| 0.10 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.82 | 0.82 | 0.82 | 0.82 | 0.82 |
| 0.20 | 0.86 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 |
| 0.30 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.87 | 0.87 | 0.87 | 0.87 | 0.87 |
| 0.40 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| 0.50 | 0.94 | 0.94 | 0.94 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.93 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 |
| 0.60 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 |
| 0.70 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 |
| 0.80 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.99 | 0.99 |
| 0.90 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 |
| 1.00 | 1.08 | 1.07 | 1.07 | 1.07 | 1.07 | 1.06 | 1.06 | 1.06 | 1.06 | 1.05 | 1.05 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 |

Table 2: Comparison of the ratio of $P^*_{A,a,c,r} / P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $f^{+}_{a-1,c,r^*}$ when $\varphi_{A,c,r}$ is fixed at 0.8, $\rho_{A,a,c,r}$ is fixed at 0.8 and $f^{+}_{A,a-1,c,r}$ is fixed at 0.5. Note $m^{+}_{A,a-1,c}$ = 0.35.

| $\rho_{A,a,c,r^*}$ | $f^{+}_{A,a-1,c,r^*}$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 |
| 0.10 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 |
| 0.20 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.87 | 0.87 | 0.87 | 0.87 | 0.87 | 0.87 |
| 0.30 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 | 0.91 |
| 0.40 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.94 |
| 0.50 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| 0.60 | 1.05 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 |
| 0.70 | 1.09 | 1.09 | 1.08 | 1.08 | 1.08 | 1.07 | 1.07 | 1.07 | 1.07 | 1.06 | 1.06 | 1.06 | 1.06 | 1.05 | 1.05 | 1.05 |
| 0.80 | 1.13 | 1.13 | 1.12 | 1.12 | 1.12 | 1.11 | 1.11 | 1.11 | 1.10 | 1.10 | 1.10 | 1.10 | 1.09 | 1.09 | 1.09 | 1.08 |
| 0.90 | 1.17 | 1.17 | 1.16 | 1.16 | 1.16 | 1.15 | 1.15 | 1.15 | 1.14 | 1.14 | 1.14 | 1.13 | 1.13 | 1.13 | 1.12 | 1.12 |
| 1.00 | 1.21 | 1.21 | 1.20 | 1.20 | 1.20 | 1.19 | 1.19 | 1.18 | 1.18 | 1.18 | 1.17 | 1.17 | 1.17 | 1.16 | 1.16 | 1.15 |

Table 3: Comparison of the ratio of $P^{*}_{A,a,c,r}/P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $f^{+}_{A,a-1,c,r^*}$ when $\varphi_{A,c,r}$ is fixed at 0.4, $\rho_{A,a,c,r}$ is fixed at 0.8 and $f^{+}_{A,a-1,c,r}$ is fixed at 0.1. Note $m^{+}_{A,a-1,c}$ = 0.35.

| $\rho_{A,a,c,r^*}$ | $f^{+}_{A,a-1,c,r^*}$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 |
| 0.10 | 0.48 | 0.48 | 0.47 | 0.46 | 0.46 | 0.45 | 0.45 | 0.44 | 0.44 | 0.43 | 0.43 | 0.43 | 0.42 | 0.42 | 0.42 | 0.42 |
| 0.20 | 0.57 | 0.55 | 0.54 | 0.52 | 0.51 | 0.50 | 0.49 | 0.48 | 0.47 | 0.47 | 0.46 | 0.46 | 0.45 | 0.45 | 0.44 | 0.44 |
| 0.30 | 0.65 | 0.63 | 0.60 | 0.58 | 0.57 | 0.55 | 0.54 | 0.52 | 0.51 | 0.50 | 0.49 | 0.48 | 0.47 | 0.47 | 0.46 | 0.46 |
| 0.40 | 0.73 | 0.70 | 0.67 | 0.65 | 0.62 | 0.60 | 0.58 | 0.56 | 0.55 | 0.53 | 0.52 | 0.51 | 0.50 | 0.49 | 0.48 | 0.47 |
| 0.50 | 0.81 | 0.78 | 0.74 | 0.71 | 0.68 | 0.65 | 0.63 | 0.61 | 0.59 | 0.57 | 0.55 | 0.54 | 0.52 | 0.51 | 0.50 | 0.49 |
| 0.60 | 0.90 | 0.85 | 0.81 | 0.77 | 0.73 | 0.70 | 0.67 | 0.65 | 0.62 | 0.60 | 0.58 | 0.57 | 0.55 | 0.54 | 0.52 | 0.51 |
| 0.70 | 0.98 | 0.93 | 0.88 | 0.83 | 0.79 | 0.75 | 0.72 | 0.69 | 0.66 | 0.64 | 0.61 | 0.59 | 0.57 | 0.56 | 0.54 | 0.53 |
| 0.80 | 1.06 | 1.00 | 0.94 | 0.89 | 0.84 | 0.80 | 0.76 | 0.73 | 0.70 | 0.67 | 0.64 | 0.62 | 0.60 | 0.58 | 0.56 | 0.55 |
| 0.90 | 1.15 | 1.08 | 1.01 | 0.95 | 0.90 | 0.85 | 0.81 | 0.77 | 0.74 | 0.70 | 0.67 | 0.65 | 0.62 | 0.60 | 0.58 | 0.57 |
| 1.00 | 1.23 | 1.15 | 1.08 | 1.01 | 0.96 | 0.90 | 0.85 | 0.81 | 0.77 | 0.74 | 0.70 | 0.68 | 0.65 | 0.63 | 0.60 | 0.58 |

Table 4: Comparison of the ratio of $P^*_{A,a,c,r}/P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $f^+_{A,a-1,c,r^*}$ when $\varphi_{A,c,r}$ is fixed at 0.8, $\rho_{A,a,c,r}$ is fixed at 0.4 and $f^+_{A,a-1,c,r}$ is fixed at 0.1. Note $m^+_{A,a-1,c}$ = 0.35.

| $\rho_{A,a,c,r^*}$ | $f^+_{A,a-1,c,r^*}$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 |
| 0.10 | 0.86 | 0.85 | 0.85 | 0.84 | 0.84 | 0.83 | 0.83 | 0.83 | 0.82 | 0.82 | 0.82 | 0.82 | 0.82 | 0.82 | 0.81 | 0.81 |
| 0.20 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.87 | 0.86 | 0.85 | 0.85 | 0.84 | 0.84 | 0.84 | 0.83 | 0.83 | 0.83 | 0.82 |
| 0.30 | 0.97 | 0.95 | 0.94 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.87 | 0.86 | 0.86 | 0.85 | 0.85 | 0.84 | 0.84 |
| 0.40 | 1.02 | 1.00 | 0.98 | 0.96 | 0.95 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.87 | 0.86 | 0.85 | 0.85 |
| 0.50 | 1.08 | 1.05 | 1.03 | 1.00 | 0.99 | 0.97 | 0.95 | 0.94 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.88 | 0.87 | 0.86 |
| 0.60 | 1.13 | 1.10 | 1.07 | 1.05 | 1.02 | 1.00 | 0.98 | 0.96 | 0.95 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 |
| 0.70 | 1.19 | 1.15 | 1.12 | 1.09 | 1.06 | 1.03 | 1.01 | 0.99 | 0.97 | 0.96 | 0.94 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 |
| 0.80 | 1.24 | 1.20 | 1.16 | 1.13 | 1.10 | 1.07 | 1.04 | 1.02 | 1.00 | 0.98 | 0.96 | 0.95 | 0.93 | 0.92 | 0.91 | 0.90 |
| 0.90 | 1.30 | 1.25 | 1.21 | 1.17 | 1.13 | 1.10 | 1.07 | 1.05 | 1.02 | 1.00 | 0.98 | 0.97 | 0.95 | 0.94 | 0.92 | 0.91 |
| 1.00 | 1.35 | 1.30 | 1.25 | 1.21 | 1.17 | 1.14 | 1.10 | 1.07 | 1.05 | 1.02 | 1.00 | 0.98 | 0.97 | 0.95 | 0.94 | 0.92 |

Table 5: Comparison of the ratio of $P^{*}_{A,a,c,r} / P_{A,c}$ for a range of values for $\varphi_{A,c,r}$ and $\rho_{A,a,c,r}$ when $\rho_{A,a,c,r^*}$ is fixed at 0.2, $f^{+}_{A,a-1,c,r}$ is fixed at 0.1, and $f^{+}_{A,a-1,c,r^*}$ is fixed at 0.05. Note $m^{+}_{A,a-1,c} = 0.35$.

| $\varphi_{A,c,r}$ | $\rho_{A,a,c,r}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 0.10 | 1.99 | 1.05 | 0.73 | 0.57 | 0.48 | 0.42 | 0.37 | 0.34 | 0.31 |
| 0.20 | 1.88 | 1.04 | 0.76 | 0.62 | 0.54 | 0.48 | 0.44 | 0.41 | 0.39 |
| 0.30 | 1.77 | 1.04 | 0.79 | 0.67 | 0.59 | 0.55 | 0.51 | 0.48 | 0.46 |
| 0.40 | 1.66 | 1.03 | 0.82 | 0.72 | 0.65 | 0.61 | 0.58 | 0.56 | 0.54 |
| 0.50 | 1.55 | 1.03 | 0.85 | 0.76 | 0.71 | 0.68 | 0.65 | 0.63 | 0.62 |
| 0.60 | 1.44 | 1.02 | 0.88 | 0.81 | 0.77 | 0.74 | 0.72 | 0.71 | 0.69 |
| 0.70 | 1.33 | 1.02 | 0.91 | 0.86 | 0.83 | 0.81 | 0.79 | 0.78 | 0.77 |
| 0.80 | 1.22 | 1.01 | 0.94 | 0.91 | 0.88 | 0.87 | 0.86 | 0.85 | 0.85 |
| 0.90 | 1.11 | 1.01 | 0.97 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 |

Table 6: Comparison of the ratio of $P^{*}_{A,a,c,r} / P_{A,c}$ for a range of values for $\varphi_{A,c,r}$ and $\rho_{A,a,c,r}$ when $\rho_{A,a,c,r^*}$ is fixed at 0.6, $f^{+}_{A,a-1,c,r}$ is fixed at 0.1, and $f^{+}_{A,a-1,c,r^*}$ is fixed at 0.05. Note $m^{+}_{A,a-1,c} = 0.35$.

| $\varphi_{A,c,r}$ | $\rho_{A,a,c,r}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 0.10 | 5.78 | 2.94 | 1.99 | 1.52 | 1.24 | 1.05 | 0.91 | 0.81 | 0.73 |
| 0.20 | 5.25 | 2.72 | 1.88 | 1.46 | 1.21 | 1.04 | 0.92 | 0.83 | 0.76 |
| 0.30 | 4.72 | 2.51 | 1.77 | 1.40 | 1.18 | 1.04 | 0.93 | 0.85 | 0.79 |
| 0.40 | 4.18 | 2.29 | 1.66 | 1.35 | 1.16 | 1.03 | 0.94 | 0.87 | 0.82 |
| 0.50 | 3.65 | 2.08 | 1.55 | 1.29 | 1.13 | 1.03 | 0.95 | 0.89 | 0.85 |
| 0.60 | 3.12 | 1.86 | 1.44 | 1.23 | 1.10 | 1.02 | 0.96 | 0.92 | 0.88 |
| 0.70 | 2.59 | 1.65 | 1.33 | 1.17 | 1.08 | 1.02 | 0.97 | 0.94 | 0.91 |
| 0.80 | 2.06 | 1.43 | 1.22 | 1.12 | 1.05 | 1.01 | 0.98 | 0.96 | 0.94 |
| 0.90 | 1.53 | 1.22 | 1.11 | 1.06 | 1.03 | 1.01 | 0.99 | 0.98 | 0.97 |

Table 7: Comparison of the ratio of $P^{*}_{A,a,c,r}/P_{A,c}$ for a range of values for $\varphi_{A,c,r}$ and $\rho_{A,a,c,r}$ when $\rho_{A,a,c,r^*}$ is fixed at 0.6, $f^{+}_{A,a-1,c,r}$ is fixed at 0.4, and $f^{+}_{A,a-1,c,r^*}$ is fixed at 0.05. Note $m^{+}_{A,a-1,c}$ = 0.35.

| $\varphi_{A,c,r}$ | $\rho_{A,a,c,r}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 0.10 | 7.76 | 3.93 | 2.65 | 2.02 | 1.63 | 1.38 | 1.19 | 1.06 | 0.95 |
| 0.20 | 7.01 | 3.61 | 2.47 | 1.90 | 1.56 | 1.34 | 1.17 | 1.05 | 0.96 |
| 0.30 | 6.26 | 3.28 | 2.29 | 1.79 | 1.49 | 1.29 | 1.15 | 1.05 | 0.96 |
| 0.40 | 5.51 | 2.95 | 2.10 | 1.68 | 1.42 | 1.25 | 1.13 | 1.04 | 0.97 |
| 0.50 | 4.76 | 2.63 | 1.92 | 1.56 | 1.35 | 1.21 | 1.11 | 1.03 | 0.97 |
| 0.60 | 4.01 | 2.30 | 1.74 | 1.45 | 1.28 | 1.17 | 1.09 | 1.03 | 0.98 |
| 0.70 | 3.25 | 1.98 | 1.55 | 1.34 | 1.21 | 1.13 | 1.06 | 1.02 | 0.98 |
| 0.80 | 2.50 | 1.65 | 1.37 | 1.23 | 1.14 | 1.08 | 1.04 | 1.01 | 0.99 |
| 0.90 | 1.75 | 1.33 | 1.18 | 1.11 | 1.07 | 1.04 | 1.02 | 1.01 | 0.99 |

Table 8: Comparison of the ratio of $P^{*}_{A,a,c,r}/P_{A,c}$ for a range of values for $\varphi_{A,c,r}$ and $\rho_{A,a,c,r}$ when $\rho_{A,a,c,r^*}$ is fixed at 0.6, $f^{+}_{A,a-1,c,r}$ is fixed at 0.05, and $f^{+}_{A,a-1,c,r^*}$ is fixed at 0.40. Note $m^{+}_{A,a-1,c}$ = 0.35.

| $\varphi_{A,c,r}$ | $\rho_{A,a,c,r}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 0.10 | 3.91 | 2.00 | 1.37 | 1.05 | 0.86 | 0.73 | 0.64 | 0.58 | 0.52 |
| 0.20 | 3.58 | 1.89 | 1.33 | 1.05 | 0.88 | 0.76 | 0.68 | 0.62 | 0.58 |
| 0.30 | 3.26 | 1.78 | 1.29 | 1.04 | 0.89 | 0.79 | 0.72 | 0.67 | 0.63 |
| 0.40 | 2.94 | 1.67 | 1.25 | 1.03 | 0.91 | 0.82 | 0.76 | 0.72 | 0.68 |
| 0.50 | 2.61 | 1.56 | 1.20 | 1.03 | 0.92 | 0.85 | 0.80 | 0.76 | 0.73 |
| 0.60 | 2.29 | 1.45 | 1.16 | 1.02 | 0.94 | 0.88 | 0.84 | 0.81 | 0.79 |
| 0.70 | 1.97 | 1.33 | 1.12 | 1.02 | 0.95 | 0.91 | 0.88 | 0.86 | 0.84 |
| 0.80 | 1.65 | 1.22 | 1.08 | 1.01 | 0.97 | 0.94 | 0.92 | 0.91 | 0.89 |
| 0.90 | 1.32 | 1.11 | 1.04 | 1.01 | 0.98 | 0.97 | 0.96 | 0.95 | 0.95 |

Table 9: Comparison of the ratio of $P^*_{A,a,c,r}/P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $\rho_{A,a,c,r}$ when $\varphi_{A,c,r}$ is fixed at 0.8, $f^+_{A,a-1,c,r}$ is fixed at 0.1, and $f^+_{A,a-1,c,r^*}$ is fixed at 0.05. Note $m^+_{A,a-1,c} = 0.35$.

| $\rho_{A,a,c,r}$ | $\rho_{A,a,c,r^*}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| 0.10 | 1.01 | 1.22 | 1.43 | 1.64 | 1.85 | 2.06 | 2.27 | 2.48 | 2.69 | 2.90 |
| 0.20 | 0.91 | 1.01 | 1.12 | 1.22 | 1.33 | 1.43 | 1.54 | 1.64 | 1.75 | 1.85 |
| 0.30 | 0.87 | 0.94 | 1.01 | 1.08 | 1.15 | 1.22 | 1.29 | 1.36 | 1.43 | 1.50 |
| 0.40 | 0.85 | 0.91 | 0.96 | 1.01 | 1.06 | 1.12 | 1.17 | 1.22 | 1.27 | 1.33 |
| 0.50 | 0.84 | 0.88 | 0.93 | 0.97 | 1.01 | 1.05 | 1.09 | 1.14 | 1.18 | 1.22 |
| 0.60 | 0.84 | 0.87 | 0.91 | 0.94 | 0.98 | 1.01 | 1.05 | 1.08 | 1.12 | 1.15 |
| 0.70 | 0.83 | 0.86 | 0.89 | 0.92 | 0.95 | 0.98 | 1.01 | 1.04 | 1.07 | 1.10 |
| 0.80 | 0.83 | 0.85 | 0.88 | 0.91 | 0.93 | 0.96 | 0.98 | 1.01 | 1.04 | 1.06 |
| 0.90 | 0.82 | 0.85 | 0.87 | 0.89 | 0.92 | 0.94 | 0.96 | 0.99 | 1.01 | 1.03 |
| 1.00 | 0.82 | 0.84 | 0.86 | 0.88 | 0.91 | 0.93 | 0.95 | 0.97 | 0.99 | 1.01 |

Table 10: Comparison of the ratio of $P^*_{A,a,c,r}/P_{A,c}$ for a range of values for $\rho_{A,a,c,r^*}$ and $\rho_{A,a,c,r}$ when $\varphi_{A,c,r}$ is fixed at 0.4, $f^+_{A,a-1,c,r}$ is fixed at 0.1, and $f^+_{A,a-1,c,r^*}$ is fixed at 0.05. Note $m^+_{A,a-1,c} = 0.35$.

| $\rho_{A,a,c,r}$ | $\rho_{A,a,c,r^*}$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| 0.10 | 1.03 | 1.66 | 2.29 | 2.92 | 3.55 | 4.18 | 4.82 | 5.45 | 6.08 | 6.71 |
| 0.20 | 0.72 | 1.03 | 1.35 | 1.66 | 1.98 | 2.29 | 2.61 | 2.92 | 3.24 | 3.55 |
| 0.30 | 0.61 | 0.82 | 1.03 | 1.24 | 1.45 | 1.66 | 1.87 | 2.08 | 2.29 | 2.50 |
| 0.40 | 0.56 | 0.72 | 0.87 | 1.03 | 1.19 | 1.35 | 1.50 | 1.66 | 1.82 | 1.98 |
| 0.50 | 0.53 | 0.65 | 0.78 | 0.90 | 1.03 | 1.16 | 1.28 | 1.41 | 1.54 | 1.66 |
| 0.60 | 0.51 | 0.61 | 0.72 | 0.82 | 0.93 | 1.03 | 1.14 | 1.24 | 1.35 | 1.45 |
| 0.70 | 0.49 | 0.58 | 0.67 | 0.76 | 0.85 | 0.94 | 1.03 | 1.12 | 1.21 | 1.30 |
| 0.80 | 0.48 | 0.56 | 0.64 | 0.72 | 0.79 | 0.87 | 0.95 | 1.03 | 1.11 | 1.19 |
| 0.90 | 0.47 | 0.54 | 0.61 | 0.68 | 0.75 | 0.82 | 0.89 | 0.96 | 1.03 | 1.10 |
| 1.00 | 0.46 | 0.53 | 0.59 | 0.65 | 0.72 | 0.78 | 0.84 | 0.90 | 0.97 | 1.03 |